Lecture 1, Week 2
Reading assignment: A guide to the next utility. Pp. 38-57
GIS Fundamentals: pp. 175-206 (Ch. 5).

Material covered in this week contributes to the accomplishment of the following course goals:

<table>
<thead>
<tr>
<th>Goal 1:</th>
<th>Demonstrate a basic understanding of the theory behind GPS and the reasons for its use.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal 4:</td>
<td>Be able to use GPS as a navigational aid</td>
</tr>
<tr>
<td>Goal 5:</td>
<td>Be able to transfer GIS data to GPS for use in the field</td>
</tr>
</tbody>
</table>

Information obtained this week will help you further understand GPS theory. It will help you understand the impact of satellite geometry on positional accuracy. It will also form the basis for use of a GPS receiver as a navigational aid and as a tool to develop waypoints for use in future navigation activities.

After studying class notes and reading assignments, participating in class discussions, and conducting lab 2, you will be able to:

- Describe the necessary arrangement and formatting of a text file that is to be used to create a waypoint file.
- Use PathFinder Office to load waypoint files to the GEOXT units.
- Use the GEOXT GPS units and the map or navigation screens to navigate to waypoints in the field.
- Use a GEOXT without real-time correction to create waypoints in the field.
- Describe the variables contained in the 4 Pythagorean equations that represent the GPS solution for the rover position.
- Explain how the Pythagorean theorem, which is typically used in two dimensions can be used in 3 dimensions.
- Describe Selective Availability and the magnitude of its impact on GPS positioning.
- Explain
  - PDOP
  - Error Budgets
  - User Equivalent Ranging Error
  - Multi-path error
Take out a piece of paper & answer the following questions:

1) What is (describe) GPS ALMANAC data?

2) What are all the purposes and/or uses of ALMANAC data?

3) Describe GPS EPHEMERIS data and how is it different from ALMANAC?

4) What are all the purposes and/or uses of EPHEMERIS data?

5) What are the mental processes necessary to allow you to navigate from here to your next class? (without a GPS)

Please confer with your TABLE-MATES to best answer these questions.
Requirements When Navigating With Compass & Pacing

1) Need to know where you are (MAP)
2) Need to know where you want to go (MAP)
3) Need something to point you in right direction (COMPASS & BEARING)
4) Nice to travel as the crow flies, but is rarely possible over land
5) Often have to break the trip into segments
   - Turn left at Grand Avenue (WAYPOINT) then turn right on Main Street (WAYPOINT)
   - Or, turn right on path ahead (WAYPOINT) to access bridge to cross river (WAYPOINT)
6) Need some way to estimate the distance you have traveled (MAP SCALE & PACING)

**SCALES** → 1:24,000 vs. 1:50,000
Which is the larger scale?
Requirements When Navigating With GPS

1) Need to know where you are (GPS)

2) Need to know where you want to go (Topo, GoogleEarth, GIS map, book)

3) Need something to point you in right direction (GPS)

4) Nice to travel as the crow flies → rarely possible...

5) Break the trip into segments....
   - Turn right at Pammel Dr. (WAYPOINT) then turn left at Stange Rd. (WAYPOINT) etc.
   - Or, turn right on path ahead (WAYPOINT) to access bridge to cross river (WAYPOINT)

6) Need some way to estimate the distance you have traveled (GPS)
WAYPOINT...

Is any single, specific location...

Your house
Starbucks
This classroom
Lamp post
Large rock along a trail
Road intersection
PLS Section corner
Orange flag tied onto a tree

....that physically identifies where you want to go, or helps you navigate to some other specific location.

For **GPS**, it is all the above, but with a **specific X, Y coordinate**
- UTM, State Plane, Longitude Latitude, etc.

"House",446527.7, 4653333.2
"StarBucks",446528.7, 4653358.2
"Intersection",446647.2, 4653355.2
"Rock By Trail",446647.2, 4653376.7
"Orange Flag",446716.7, 4653373.7
"Sci II Rm 233",446715.2, 4653392.7
Can be gotten off Topoquads by hand as we did in Lab

Note Declination ...Sullivan Line
From ArcMap / GIS
Or, from other GIS software and imagery.
Apps, Books, and other Software
Lecture 2, Week 2
COORDINATES FOR THE SAME

<table>
<thead>
<tr>
<th></th>
<th>Easting</th>
<th>Northing</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD27</td>
<td>444000</td>
<td>4653000</td>
</tr>
<tr>
<td>NAD83</td>
<td>443984</td>
<td>4653210</td>
</tr>
</tbody>
</table>

Read NAD27 from a map for navigation with a GPS set to NAD83

You will wind up 16 meter NORTH of the point, 210 m WEST of the point
(± Map Accuracy error...up to 20.32m 1/30th of an inch)

16 -210

Read NAD83 from a map for navigation with a GPS set to NAD27

You will wind up 16 meter SOUTH of the point, 210 m EAST of the point
(± Map Accuracy error 20.32m)

-16 210

Collect NAD83 locations with GPS and plot to a NAD27 map

You will wind up 16 meter SOUTH of the point, 210 m EAST of the point

Collect NAD27 locations with GPS and plot to a NAD28 map

You will wind up 16 meter NORTH of the point, 210 m WEST of the point
How Do We Know The Positions of The SV’s

- Omnidirectional signal sent by the SV is encoded with a **NAVIGATION MESSAGE** which can be read by the GPS receiver.

- **Navigational message** includes orbital parameters called **BROADCAST EPHEMERIS** from which the GPS receivers can compute SV coordinates \((X, Y, Z)\).
  - These are **Cartesian coordinates** in a geocentric system (WGS-84)
  - Origin at the Earth center of mass: **Earth Centered Earth Fixed (ECEF)**

- Algorithm that transforms the orbit parameters into WGS-84 satellite coordinates is called the **EPHEMERIS ALGORITHM**
How Do We Know The Positions of The SV’s

- The time that the signal is transmitted from the satellite is encoded on the signal... using the time according to the **atomic clock onboard the SV**.

- Time of signal reception is recorded by **GPS receiver; uses clock synchronized to the SV atomic clocks**.

- The difference in these times is called the **PSEUDORANGE**

  \[ \text{PSEUDORANGE} = (\text{time difference}) \times (\text{speed of light}) \]

  **PSEUDO**range because it includes **CLOCK ERROR** (given in the NAV MESSAGE)

- The **unknown GPS receiver clock error** can be estimated by the user as well as the **unknown GPS receiver coordinates**.

- There are **4 unknowns** → hence we need a minimum of **4 pseudorange measurements**...
GPS Pseudorange Observation \( \rightarrow \) SV & Receiver Clocks

\[ X = 1 1 1 1 \rightarrow 1 1 1 1 -1 -1 1 1 -1 -1 -1 1 1 1 1 1 1 -1 -1 \]

\[ \uparrow \uparrow \quad \uparrow \uparrow \quad X \]

\[ 1 1 1 1 -1 -1 1 1 -1 -1 -1 1 1 1 1 1 1 -1 -1 \]

\[ \uparrow \uparrow \quad \uparrow \uparrow \quad = \quad 1 1 -1 -1 -1 1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1 \]

\[ \rightarrow \text{MIS-MATCH} \]
GPS Position Solution...

\[(X_1-U_x)^2 + (Y_1-U_y)^2 + (Z_1-U_z)^2 = (SL(T_1-C_b))^2\]
\[(X_2-U_x)^2 + (Y_2-U_y)^2 + (Z_2-U_z)^2 = (SL(T_2-C_b))^2\]
\[(X_3-U_x)^2 + (Y_3-U_y)^2 + (Z_3-U_z)^2 = (SL(T_3-C_b))^2\]
\[(X_4-U_x)^2 + (Y_4-U_y)^2 + (Z_4-U_z)^2 = (SL(T_4-C_b))^2\]

\(X_i, Y_i, Z_i\) are coordinates for the \(i\)th satellite
\(U_x, U_y, U_z\) are rover coordinates to be determined

\(SL\) is the speed of light (186,282.4 miles sec\(^{-1}\) or 299,792,458 m sec\(^{-1}\))

\(T_i\) is the time necessary to receive a signal from the \(i\)th satellite

\(C_b\) is the clock bias of the rover (~1/9,000,000,000\(^{th}\) of a second)

From **Pythagoras Theorem** (and a least squares adjustment to minimize range errors) the GPS receiver estimates its ....

**Unique Position**
$C^2 = X^2 + Y^2 \quad S^2 = C^2 + Z^2$

$\therefore \quad S^2 = X^2 + Y^2 + Z^2$
Satellite Geometry & DOP

The effect of satellite geometry on position error is called **Dilution of Precision → DOP**

A GPS receiver computes DOP constantly....using the best 4 visible SV’s

The better the geometry (*proper spread across sky*) the LOWER the DOP

**DOP** affected by → **User Equivalent Range Error (UERE)**

- Any error contributing to the **error budget** of receiver’s positioning
- Expressed as an **equivalence** in the **range between the receiver and SV**
- **Originates from different, independent sources:**
  - A prediction of maximum anticipated total **UERE** (minus ionospheric error) is provided in each satellite's navigation message as the **user range accuracy (URA)**.

**Best** geometry = 3 SV’s low in sky (120° apart) with 4th SV over head

**Worst** = 4 SV’s bunched together or in a straight line in the sky

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} E_i}$$
### The GPS Error Budget: Independent Errors

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncorrected Error Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionosphere</td>
<td>0-30 m</td>
</tr>
<tr>
<td>Troposphere</td>
<td>0-30 m</td>
</tr>
<tr>
<td>Measurement Noise</td>
<td>0-10 m</td>
</tr>
<tr>
<td>Ephemeris Data</td>
<td>1-5 m</td>
</tr>
<tr>
<td>Clock Drift</td>
<td>0-1.5 m</td>
</tr>
<tr>
<td>Multipath</td>
<td>0-1 m</td>
</tr>
<tr>
<td>Selective Availability</td>
<td>0-70 m</td>
</tr>
</tbody>
</table>

Atmosphere (Iono & Trop) refract GPS signal ➔ speed different than in space’s vacuum

- $(\text{SIGNAL SPEED} \times \text{TIME})_{\text{space}} \neq (\text{SIGNAL SPEED} \times \text{TIME})_{\text{atmos.}}$ ➔ ERROR
- Ephemeris/Clock Drift/Measurement Noise
- GPS signals contain info about:
  - Ephemeris (orbital position) errors
  - Rate of clock drift for each SV
- Ephemeris error data may not exactly model the true SV motion or rate of drift
- Signal distortion by measurement noise
Satellite Geometry & various DOP measurements

GDOP (Geometric) → PDOP (3D Position) → TDOP (Time)
HDOP (Horizontal) → VDOP (Vertical)
NDOP (North) → EDOP (East)

- Can’t assume that differential correction will take out all errors (significant or not)
  1) especially multipath errors
  2) Or errors caused by high PDOP
- But, DOP is calculated by the receiver, so the software offers filters to prevent operation when DOP is too high...TRIMBLE

\[ PDOP = \sqrt{HDOP^2 + VDOP^2} \]
Manually Calculating Dilution of Precision

SV coordinates in ECEF XYZ from Ephemeris Parameters and SV Time (i = 0...3)

<table>
<thead>
<tr>
<th>i</th>
<th>SVx_i</th>
<th>SVy_i</th>
<th>SVz_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15524471.175</td>
<td>-16649826.222</td>
<td>13512272.387</td>
</tr>
<tr>
<td>1</td>
<td>-2304058.534</td>
<td>-23287906.465</td>
<td>11917038.105</td>
</tr>
<tr>
<td>2</td>
<td>16680243.357</td>
<td>-3069625.561</td>
<td>20378551.047</td>
</tr>
<tr>
<td>3</td>
<td>14799931.395</td>
<td>-21425358.24</td>
<td>6069947.224</td>
</tr>
</tbody>
</table>

Receiver Position Range Estimate in ECEF XYZ \( \rightarrow \) with position error

\[
Rx = -730000 \quad Ry = -5440000 \quad Rz = 3230000
\]

Receiver Position Range Estimates to SV\(_i \rightarrow R_i\)

\[
R_i = \sqrt{(SVx_i - Rx)^2 + (SVy_i - Ry)^2 + (SVz_i - Rz)^2}
\]

Compute Directional Derivatives for XYZ and Time for each of the 4 SVs

\[
\begin{align*}
Dx_i &= \frac{SVx_i - Rx}{R_i} \\
Dy_i &= \frac{SVy_i - Ry}{R_i} \\
Dz_i &= \frac{SVz_i - Rz}{R_i} \\
Dt_i &= -1
\end{align*}
\]

Solve for correction to Receiver Position Estimates

\[
A = \begin{bmatrix}
Dx_0 & Dy_0 & Dz_0 & Dt_0 \\
Dx_1 & Dy_1 & Dz_1 & Dt_1 \\
Dx_2 & Dy_2 & Dz_2 & Dt_2 \\
Dx_3 & Dy_3 & Dz_3 & Dt_3
\end{bmatrix}
\]

\[
P = (A^T \cdot A)^{-1} = \begin{bmatrix}
P_{0.0} & & & \\
& P_{1.1} & & \\
& & P_{2.2} & \\
& & & P_{3.3}
\end{bmatrix}
\]

Compute Geometric Dilution of Precision (GDOP) terms:

\[
GDOP = \sqrt{P_{0.0} + P_{1.1} + P_{2.2} + P_{3.3}} \quad GDOP = 6.806
\]

\[
PDOP = \sqrt{P_{0.0} + P_{1.1} + P_{2.2}} \quad PDOP = 6.717
\]

\[
TDOP = \sqrt{P_{3.3}} \quad TDOP = 2.871
\]
<table>
<thead>
<tr>
<th>Type of DOP</th>
<th>Interpretation</th>
<th>Coordinates Involved</th>
<th>Typical Interested User</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDOP</td>
<td>Geometrical</td>
<td>Ux, Uy, Uz, Ut (3-D + Time)</td>
<td>Mostly theoretical interest users</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Moving time-sync</td>
</tr>
<tr>
<td>HDOP</td>
<td>Horizontal</td>
<td>Ux, Uy (local horiz. coords)</td>
<td>Maritime users</td>
</tr>
<tr>
<td>VDOP</td>
<td>Vertical</td>
<td>Uz (Altitude)</td>
<td>Air-related users</td>
</tr>
<tr>
<td>TDOP</td>
<td>Time</td>
<td>Ut (Time)</td>
<td>Time Synchronization</td>
</tr>
</tbody>
</table>
Multipath

-Propagation phenomenon that results in radio signals reaching the receiver antenna by two or more paths.

Sources/Causes:
- Atmospheric ducting
- Ionosphere reflection or refraction
- Reflection from:
  - Water bodies
  - Mountains
  - Trees & Buildings

- High-end receiver/antenna combinations are more robust in rejecting multipath

- Consumer-level receivers tolerate a high amount of multi-path

- Differential correction will not compensate for multipath errors
Mitigating Multipath

- Avoid collecting data near big/tall structures (if possible)
- Avoid collecting data under dense canopy conditions (if possible)
- Used remote antennas
- Choke ring antennas – notable for their ability to reject multipath signals
Error... and Probability Density Function (PDF)

PDF, \( f(x) \), of a continuous random variable (CRV)...

The Probability Density Function (PDF) can be integrated to get the probability that the CRV takes a value in a given interval.

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

Accuracy vs. Precision

Bias vs Spread

2D Error

Uncorrected GPS Data \( \rightarrow \) 1 True Location

Mean (\( \mu \)) ≠ True
PDF can be integrated to get a probability that a CRV takes a value in a given interval

\[ \mu \text{ or } \bar{X} \text{ or } m = \text{Mean} = \frac{\Sigma x}{N} \]

\[ \sigma = \text{Standard Deviation} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2} \]

\[ \pi = 3.14159 \]

\[ e \text{ or } \exp = 2.71828 \] (base of the natural logarithm)

Distance Error \(\rightarrow\) Measured - True

RMSE = Root Means Squared Error = \(\sqrt{\frac{1}{N} \sum_{i=1}^{N} (distance \ errors)^2}\)

<table>
<thead>
<tr>
<th>Distance Errors</th>
<th>(Distance Error)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.2</td>
<td>125.44</td>
</tr>
<tr>
<td>12.3</td>
<td>151.29</td>
</tr>
<tr>
<td>14.1</td>
<td>198.81</td>
</tr>
<tr>
<td>13.6</td>
<td>+ 184.96</td>
</tr>
</tbody>
</table>
|                 | 660.5 / 4 = 165.125 \(\rightarrow\) \(\sqrt{165.125} = 12.85009728\) RMSE

So, what is the probability of an error being \(\leq\) 10m?

\[(\text{Dist/RMSE})^2 = (10/\text{12.85})^2 = 0.6056\]

\[P(\text{error} \leq \text{Dist}) = 1 - e^{-0.6056} = 0.45 \rightarrow 45\% \text{ chance error} < \text{10m}\]