Map Projections

The GLOBE

ADVANTAGES
• Directions  True
• Distances  True
• Shapes  True
• Area  True

DISADVANTAGES
• Very small scale with little detail.
• Costly to reproduce and update.
• Difficult to carry around.
• Bulky to store.

FACTS
• Parallels are parallel & equally spaced on meridians
• Meridians (& arcs of Great Circles) are straight lines (If viewed perpendicular to Earth’s surface)
• Meridians equally spaced on parallels, but converge at poles & diverge toward the equator
• Meridian spacing = Parallel spacing at Equator
• Meridians @ 60° are ½ as far apart as parallels
• The scale factor at each point is the same in any direction

Great circles
• Disks with centers at Earth’s center
• The shortest 2-point distance on a sphere

Rhumb Lines (loxodromes)
• Cross all meridians of LON @ same angle
• Walking along a fixed bearing (e.g., N45°E)
Great Circle

- Shortest distance between two points on Earth’s surface.
- Any slice of a plane through a sphere that intersects the sphere’s center point
- Equator & all lines of Longitude

Great Circle Distance

Consider two points on the Earth’s surface, A with geographic coordinates (lat, lon), \((\phi_A, \lambda_A)\), and B, with geographic coordinates \((\phi_B, \lambda_B)\).

The great circle distance from point A to point B is given by the formula:

\[
d = r \cdot \cos^{-1}\left[(\cos(\phi_A)\cos(\phi_B)\cos(\lambda_A - \lambda_B) + \sin(\phi_A)\sin(\phi_B))\right],
\]

where d is the shortest distance on the surface of the Earth from A to B, and r is the Earth’s radius, approximately 6378 km.

This formula may be used to find the distance distortion caused by a projection between two points, for example, between Ursine and Moab, Utah, when using UTM Zone 12N coordinates, NAD83?

Great circle distance:
Latitude, longitude of Ursine, Utah = 37.98481°, -114.216944°
Latitude, longitude of Moab, Utah = 38.57361°, -109.551111°

\[
d = 6378 \cdot \cos^{-1}\left[(\cos(37.98481)\cos(38.57361)\cos(-114.216944 - 109.551111) + \sin(37.98481)\sin(38.57361))\right] \\
= 412,906 \text{ km}
\]

Grid distance (UTM Zone 12N coordinates):
Grid coordinates of Ursine, Utah = 217,529.8, 4,208,972.8
Grid coordinates of Moab, Utah = 626,239.2, 4,270,405.9

\[
d_g = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}^{0.5} \\
= \sqrt{(217,529.8 - 626,239.2)^2 + (4,208,972.8 - 4,270,405.9)^2}^{0.5} \\
= 413,300 \text{ km}
\]

distortion is 412,906 - 413,300 = -0.394 km, or a 394 meter lengthening
Rhumb Line (Loxodrome)

Line of constant bearing (e.g., 292.5°)
Map Projections

Map Projection Definition ➔ Mathematical function relating features on a curved surface to features on a plane

**NEEDED BECAUSE:**

1) Lat (φ), Lon (λ), & angular measures have computational problems
   - Distance ➔ Units (NS ≠ EW)
   - Direction ➔ Great Circles vs. Rhumb Lines
   - Area ➔ Measures complicated
   - Display ➔ Problems (Spherical → Flat)

2) Need X & Y rectangular/grid (Cartesian) coordinates

3) Ellipsoid is not a developable surface
   - No part of it can be cut out and laid flat

**Developable Surfaces:**
- CONE
- CYLINDER
- PLANE
- HYBRIDS (cone + cylinder)
Map Projections

“Ideal” Projection Would Preserve:
- Length of lines
- Direction of lines
- True Size of Features
- True Shape of Features

“Real” Projections are a Compromise:
- Maintain (as nearly as possible) one or more of the desired properties at the expense of the others

Classes of Map Projections:

- **Equal Area** – preserves area at the expense of shape
- **Conformal** – preserves shape at expense of area
- **Equidistant** – preserves distances
- **Azimuthal** – preserves direction of lines (not as exclusive as the others)
  - Azimuthal Equal Area
  - Azimuthal Equidistant
Distortion are Unavoidable

True Scale: **Secant Projection Case**

Unequal Differences in Scale:

- AB < A’B’ → A’B’ scale > 1
- BC < B’C’ → B’C’ scale > 1
  - But → A’B’ ≠ B’C’
- CD > C’D’ → C’D’ scale < 1
  - And → B’C’ ≠ C’D’

To Approximate Scale Factor

Compare...
- Great Distance  AB &
- GRID XY Distance  A’B’

Center of Ellipsoid and Projection Source

Scale factor < 1
Scale factor = 1
Scale factor > 1
Different Projections Distort in Different Ways

Affected by....
- Projection Source (imaginary light)
  - Orthographic
  - Stereographic
  - Gnomonic
- Orientation of Map Surface
  - Tangent
  - Secant

Planar Maps
- Tangent $\rightarrow$ 1 point of true scale
- Secant $\rightarrow$ Circle of true scale

Cylindrical Maps
- Tangent $\rightarrow$ 1 meridian of true scale
- Secant $\rightarrow$ 2 meridians of true scale

Conical Maps
- Tangent $\rightarrow$ 1 parallel of true scale
- Secant $\rightarrow$ 2 circles of true scale
Geometric Properties Preserved & Distorted

4 Distortions

- Size
- Angle
- Distance
- Direction

Map projections that avoid 1 or more of these are said to preserve certain properties of the globe

**Equivalence** → So called Equal-Area
- Maintain correct proportion in sizes of areas (but, scales differ globe → grid)
- Ellipses in Cylindrical Equal-Area are distorted... but the areas are maintained

**Conformality** → Same shape
- Circles distorted in size, but not shape
- Angles measured anywhere can be plotted without distortion
- Good for large scale surveying & mapping
- Transverse Mercator, Lambert Conformal Conic, Polar Stereographic

**Equidistance** → Distance measures accurate along straight lines...
- From 1 or 2 points only
- Ellipses on Cylindrical Equidistant map vary in size & shape
- N-S axis of each ellipse is same length & true scale along meridians

**Azimuthality** → Preserve directions
- From 1 or 2 points to all other points on the map
- Ellipses on Gnomonic map vary in size & shape, but all orient toward center
  - And, is the true direction
Mercator – Cylindrical Conformal (Angles & Shapes)

Gall Peters – Cylindrical Equal Area
Map Projections

True Scale: **Tangent Projection Case**

- **Conical**
- **Cylindrical**
- **Azimuthal**

1 LAT of True Scale "**Standard Circle**"

Scale Factor = 1 or (1:1)

1 Point of True Scale
Map Projections

True Scale: **Secant Projection Case**

- **Conical**
  - Now, each has 2 LATs of True Scale

- **Cylindrical**
  - Now, 1 LAT of True Scale, rather than a single point

- **Azimuthal**

**Standard Circles**
Projection Names

Reflect the developed surface...
- Conical (cone)
- Cylindrical (cylinder)
- Azimuthal (plane)

And, Orientation...
- Cylindrical surface with axis = polar axis
  EQUATORIAL $\rightarrow$ tangent line = equator or secants parallel to equator

- Cylinder with tangent perpendicular to equator
  TRANSVERSE $\rightarrow$ tangent line or central meridian on line of longitude

Odd-ball not based on the 3 basic shapes...
- Pseudocylindrical (Mollweide)
- Sinusoidal
- Goode Homolosine
- HEALPix
Projection Parameters

MUST SPECIFY FOR EACH......

Azimuthal (plane)
- Projection center (imaginary light source)
- Orientation of plane (with ref. to light source)
- Tangent vs. standard circle intersection secant

Conical (cone)
- Size & Orientation of cone
- Points of intersections
  - Standard parallels
  - Central Meridian
  - Origin

Cylindrical (cylinder)
- Central Meridian
- Origin
Three Measures of NORTH?

Magnetic

Geographic

Grid North ???
Common Map Projections

Lambert Conformal Conic (LCC)

- Cone intersects Ellipsoid in 2 places → Standard Parallels (SP’s)
  - SP’s are where distortion is least
  - SP’s chosen to enclose 2/3 of N-S map area
  - Distortion between SP’s uniformly lower than outside SP’s
  - Distortion increasingly complex away from SP’s

- Meridians converging straight lines (off the map extent)

- Parallels are concentric circles

- Meridian & Parallels cross at right angles
  - An essential of Conformality → preserves angles →
  - FAA sectional aeronautical charts are LCC
    → Good Direction & Shape relationships

- LCC best for large E-W map extents
  - U.S.A
  - Russia
  - Montana
Common Map Projections

Mercator

- Equator tangent to the CYLINDER (or secant, 10° N & S)
- Used for Navigation or Maps of Equatorial Regions
- Any straight line is a Rhumb Line (constant Direction)
- **Directions** along a Rhumb Line are true between **any** two points on a Mercator map, but usually **not** the shortest distance between two points.

- Lines of LAT straight & parallel
- **Distances** are true **only along Equator**, but are reasonably correct within 15° N or S
- **Areas & Shapes** greatly distorted @ high LATs
- Mercator maps are considered **conformal** in that angles & shapes for “small areas” (7.5’ topo quad) are basically true
- Mercator maps are not:
  - True perspective
  - Equal Area
  - Equidistant
Nautical Navigation Chart – Mercator

LAKE MICHIGAN

Mercator Projection
Scale 1:600,000 at Lat 44°00′N
North American Datum of 1983
(World Geodetic System 1984)

Additional information can be obtained at nauticalcharts.noaa.gov

HORIZONTAL DATUM
The horizontal reference datum of this chart is North American Datum 1983 (NAD 83) and is considered equivalent to World Geodetic System 1984 (WGS 84) for practical plotting purposes. Positions referred to the North American 1927 Datum do not require conversion to NAD 83 for plotting on this chart.

SOUNDINGS IN FEET IN BLUE TINT AREAS AND IN FATHOMS ELSEWHERE

SAILING DIRECTIONS. Bearings of sailing courses are true and distances given thereon are in statute miles between points of departure.

AIDS TO NAVIGATION. Consult U.S. Coast Guard Light List for supplemental information concerning aids to navigation.

CAUTION
The natural scale of this chart varies by 7 percent from top to bottom. Graphic scales shown are accurate only for the range of latitude in closest proximity to where they are positioned.
Common Map Projections

Transverse Mercator (TM)

- Projection Origin (imaginary light) = Ellipsoid Center
- Intersects Ellipsoid at 1 tangent or 2 secant lines
  - = lines of true scale (scale factor = 1)
  - Between secants → middle line = Central Meridian (CM)
- LON lines ≈ straight & evenly spaced near tangent
- Bands of no/low distortion run N-S
- Distortion ↑ dramatically outside & away from secant lines

- Most lines of LAT & LON are curved...
  - Great Circle & Rhumb Lines curved
  - So, quadrangle sizes & shapes all different
  - Complicates point location & direction measures
  - Solution → New X, Y GRID
    - Superimposed on warped GEO projection
    - = True rectangle intersections →
    - Permits linear & angular measurements

- TM best for large N-S map extents
  - South America, Chile, Illinois

Multiple TM map extents
The UTM System

UTM coordinates are based on a family of 120 Transverse Mercator map projections (two for each UTM zone, with one for each N/S hemisphere).

- The earth is divided into 60 zones, each 6° wide in longitude (with the exception of a few non-standard-width zones for Svalbard and southwest Norway).

![UTM Zone Numbers Diagram]

- Numbering of zones begins at 180° and proceeds eastward.
  - Zone 1 is from 180°W to 174°W,
  - Zone 2 is from 174°W to 168°W, and so on.
- Each zone has a central meridian.
  - Zone 1 central meridian is 177°W,
  - Zone 2 central meridian is 171°W, and so on.
The UTM System

- The X value, called the Easting, has a value of 500,000m at the central meridian of each zone.
- The Y value, called the Northing, has a value of 0m at the equator for the northern hemisphere, 10,000,000m at the equator for the southern hemisphere.

- UTM is limited to the area between 84°N and 80°S. Beyond that, Universal Polar Stereographic (UPS) coordinates are used.
7.5 Minute Topo Quads – UTM

Produced by the United States Geological Survey
North American Datum of 1983 (NAD83)
World Geodetic System of 1984 (WGS84). Projection and
1000-meter grid: Universal Transverse Mercator, Zone 15T
10 000-foot ticks: Iowa Coordinate System of 1983
(north zone)

Imagery..................NAIP, August 2006 - September 2006
Roads.............................US Census Bureau TIGER data
with limited USGS updates, 2005
Names.............................GNIS, 2008
Hydrography..............National Hydrography Dataset, 2006
Contours..........................National Elevation Dataset, 1999

UTM GRID AND 2010 MAGNETIC NORTH DECLINATION AT CENTER OF SHEET
Common Map Projections

Polar Gnomonic

- Tangent to Ellipsoid @ N or S pole
- Projection center is often Earth center, but can vary ➔
- LAT lines are concentric rings but not equally spaced
- All LAT lines (but equator) can be shown
- LON lines are straight & radiate from tangent
- Angles of LON are true ➔ Great Circles
- Rhumb lines approximated by straight lines
- Scale factor is variable
  - Good @ tangent
  - Good within standard circle
- Polar regions are routinely mapped this way
Common Map Projections

Polyconic (PC)

- Lines projected from Earth center to series or cones
- Each cone is tangent to a parallel of LAT
- Central meridian (CM) of map extent is straight & scale factor = 1
- LAT lines are not concentric, but intersect with extension of CM
- Distances between LAT lines along CM are $\propto$ to true distances
- Distance between meridians are similarly proportioned
- Projection is neither conformal or equivalent
  - Is a compromise that minimizes all distortions
  - Both map qualities are closely approx. for small areas
State Plane System

Standard set of projections for the US
• Unique set of coords per state.
• States may 1 or more zones.
  • WI & MN → 3
  • CA → 5
  • Limits distortion errors.
• Earth assumed flat for small areas without introducing much distortion.
• Counties & multiple counties may also have their own systems.

Two basic types:
Lambert Conformal Conic (EAST-WEST)
• Parallels placed at 1/6th of zone width N-S with central meridian (points to N) near center

Transverse Mercator (NORTH-SOUTH)
• Specifies a central meridian that defines “grid north”
• All parallels of latitude & meridians are curved (except CM)
• Uses GRID system instead