Material covered in this week contributes to the accomplishment of the following course goals:

**Goal 1:** Demonstrate a basic understanding of the theory behind GPS and the reasons for its use.

**Goal 2:** Be able to use GPS as a navigational aid

Information obtained this week will help you further understand GPS theory and the sources and magnitude of positional error in GPS. It will help you understand the impact of satellite geometry on positional accuracy. It also will allow you to see the impact of real-time correction on navigational accuracy.

After studying class notes and reading assignments, participating in class discussions, and conducting lab. 3, you should be able to:

- Discuss the importance of the three sources of error on GPS positions (satellite position errors, atmospheric errors, rover errors)
- Discuss the meaning of PDOP and explain the uses of the various dilutions of precision measures.
- List situations where the other DOP’s (HDOP, VDOP, GDOP, and TDOP) would be applicable
- Discuss the impact of satellite geometry on accuracy of GPS positions.
- Describe the impact of real-time correction on navigational accuracy
- Describe the basic processes involved in post processing GPS data
- Use PFO to upload a waypoint file to the GPS rover
- Explain
  - RMSE
  - 2dms
  - Circular Error Probable (CEP)
  - Probability of error
- Given a data set, be able to calculate
  - RMSE
  - 2dms
  - Probability of error
Sources of Error on GPS Position Accuracy

• **SV Geometry**

• **UERE**
  - Ephemeris errors
  - Atmosphere
  - Receiver Errors
    Noise & Interference/Multipath

• **User Blunders**
  Incorrect Datum Selection etc.
Main Sources of Ranging Errors (UERE):

**Ephemeris data**, Errors in the transmitted location of the satellite

**Satellite clock**, Errors in the transmitted clock time, including SA if in effect

**Ionosphere**, Errors in the corrections of pseudorange caused by ionospheric effects

**Troposphere**, Errors in the corrections of pseudorange caused by tropospheric effects

**Multipath**, Errors caused by reflected signals entering the receiver antenna

**Receiver**, Errors in the receiver's measurement of range caused by thermal noise, software accuracy, and inter-channel biases
Various DOP measures:

- GDOP (Geometric)
- PDOP (3D Position)
- HDOP (Horizontal)
- VDOP (Vertical)
- TDOP (Time)
- NDOP (North)
- EDOP (East)

Differential correction will not compensate for DOP errors

But, DOP is calculated by the receiver, so the software offers filters to prevent operation when DOP is too high....TRIMBLE
## Position Accuracy Measures

<table>
<thead>
<tr>
<th>Measure</th>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1D</strong></td>
<td><strong>Root Mean Square</strong></td>
<td>RMS or RMSE The square root of the average of the squared errors. For straight line “error distances”</td>
</tr>
<tr>
<td><strong>2D</strong></td>
<td><strong>Twice Distance RMS</strong></td>
<td>2dRMS Twice the RMS of the horizontal errors</td>
</tr>
<tr>
<td><strong>2D</strong></td>
<td><strong>Circular Error Probable</strong></td>
<td>CEP A circle's radius, centered at the true antenna position, containing 50 percent of the points in the horizontal (x, y) scatter plot</td>
</tr>
<tr>
<td><strong>2D</strong></td>
<td><strong>Horizontal 95 Percent Accuracy</strong></td>
<td>R95 A circle's radius, centered at the true antenna position, containing 95 percent of the points in the horizontal (x, y) scatter plot</td>
</tr>
<tr>
<td><strong>3D</strong></td>
<td><strong>Spherical Error Probable</strong></td>
<td>SEP A sphere's radius, centered at the true antenna position, containing 50 percent of the points in the three-dimensional (x, y, z) scatter plot</td>
</tr>
</tbody>
</table>
## Position Accuracy Measures

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Accuracy measure</th>
<th>Probability (percent)</th>
<th>Typical usage (dimension)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>rms</td>
<td>68</td>
<td>vertical</td>
</tr>
<tr>
<td>2</td>
<td>CEP</td>
<td>50</td>
<td>horizontal</td>
</tr>
<tr>
<td>2</td>
<td>rms</td>
<td>63-68</td>
<td>horizontal</td>
</tr>
<tr>
<td>2</td>
<td>R95</td>
<td>95</td>
<td>horizontal</td>
</tr>
<tr>
<td>2</td>
<td>2drms</td>
<td>95-98</td>
<td>horizontal</td>
</tr>
<tr>
<td>3</td>
<td>rms</td>
<td>61-68</td>
<td>3-D</td>
</tr>
<tr>
<td>3</td>
<td>SEP</td>
<td>50</td>
<td>3-D</td>
</tr>
</tbody>
</table>
For Example: What is more accurate……a 2m CEP or a 3m RMS(3D) ??

- Go down the “RMS(3D)” column to the "CEP" row.
- The entry in this cell is 2.5.
- So, CEP = RMS(3D)/2.5 = 3/2.5 = 1.2 meters.

So, a system with a RMS(3D) of 3m will have a CEP of 1.2 ...and is, therefore, more accurate than a system with a CEP of 2m.
Most Common Error/Accuracy Measures

Circular Error Probable (50%)
radius of a circle, centered about the mean, whose boundary is expected to include 50% of the population

RMSE (68%) 1-sigma
Distance Error → Measured – True
RMSE = Root Means Squared Error = \( \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\text{distance errors})^2} \)

2dRMS (95%)
2 * RMSE

10 of 20 points
14 of 20 points
19 of 20 points
Elliptical Error

\[ \pi = 3.14159 \]

\[
\text{Area} = (\pi \times \text{Max. Length of spread} \times \text{Max. width of spread}) / 4
\]

\[
\text{Area} = (3.14159 \times 4.1\text{m} \times 2.8\text{m}) / 4
\]

\[
\text{Area} = 9.016\text{m}^2
\]
Probability of an Error of Some Distance

<table>
<thead>
<tr>
<th>ERROR (Measured – True)</th>
<th>(ERROR)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
</tr>
</tbody>
</table>

\[ N = 4 \quad \text{Sum} = 147 \]

\[ RMSE = \sqrt{\frac{\sum \text{error}^2}{N}} \]

\[ RMSE = \sqrt{\frac{147}{4}} = 6.062 \text{m} \]

So, what is the probability \((P)\) of an error of \(\leq 2\)m?

\[ P = 1 - e^{-\left(\frac{\text{Dist}}{\text{RMSE}}\right)^2} \]

\[ P = 1 - 2.718281828^{-(2/6.062)^2} \]

\[ P = 1 - 2.718281828^{-(0.329924)^2} \]

\[ P = 1 - 2.718281828^{-0.1088499} \]

\[ P = 1 - 0.896865 \Rightarrow 0.103 \Rightarrow 10.3\% \]
Differential Correction

Cooperation between 2 GPS receivers

• Stationary (Base) – On a precisely surveyed position
• Dynamic (Rover) – Moving around...unknown positions
• Base & Rover receive same SV signals @ same times

If BASE & ROVER are close → Signal errors comparable

• Rather than use timing signals to calculate ROVER position....
• BASE uses its known geographic position used to...
  • Ask......What should signal transmission time be (SV to Rover)?
  • Works the 4 Solution Equations backwards
  • Calcs ∈(SV’s signal travel times)
  • Estimates time corrections for all SV’s → doesn’t know which SV’s are being used
  • Broadcasts corrections → information message (includes Receiver’s Latency)

Cancels out most natural & human-made errors

• SV & Receiver clock errors
• Ephemeris or imprecise orbital info
• Atmospheric effects
• STUFF NOT FIXED
  • Multipath – specific to ROVER location
  • Remaining Effects or Bad SV Geometry → DOP
  • Receiver noise errors
Differential Correction

Post Processing

• Return to office with ROVER
• Use software (PFO) to select nearest BASE station

  • CORS – NGS’s Continuously Operating Ref. Station
    • http://geodesy.noaa.gov/CORS/GoogleMap/CORS.shtml
  • UNAVCO – University NAVstar COnsortium
    • Study of Plate tectonics, Glacier Movement, etc.

• IAAM on campus → not always operational
• Next closest → Slater (15km)
Differential Correction

Real-Time

- Corrections applied instantaneously while in the field
- **SBAS** – Satellite-Based Augmentation System
  - Network of 25 BASE locations
    - Ranging & Integrity Monitoring Stations (**RIMS**)
    - Known with extreme accuracy & precision (mm)
    - Receive both L1 & L2 frequencies
  - Similar to WAAS – Wide Area Augmentation System (more later)
  - **RIMS** receive standard GPS signals (including GLONASS & GALILEO)
    - determines correction parameters
    - Relays info message with these parameters back to 1 or more SV’s
    - SV’s broadcasts info message... that ROVERs pick up
  - ROVER (Set to: “**Real-time Settings**”...”**Integrated SBAS**”) interpret RIM message
- Still not good enough to serve as sole means of navigation for aviators
  - Still issues with accuracy & reliability
- Class B, C, D airports still rely heavily on **ILS** approaches → $$$$
<table>
<thead>
<tr>
<th>1D x error</th>
<th>1D y error</th>
<th>Sums of Errors²</th>
<th>Distance Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13.66</td>
<td>-10.5</td>
<td>296.8456</td>
<td>17.22921</td>
</tr>
<tr>
<td>-7.66</td>
<td>11.5</td>
<td>190.9256</td>
<td>13.81758</td>
</tr>
<tr>
<td>1.66</td>
<td>-14.5</td>
<td>213.0056</td>
<td>14.59471</td>
</tr>
<tr>
<td>7.33</td>
<td>-5.5</td>
<td>83.9789</td>
<td>9.164</td>
</tr>
<tr>
<td>6.33</td>
<td>4.5</td>
<td>60.3189</td>
<td>7.766524</td>
</tr>
<tr>
<td>6.33</td>
<td>15</td>
<td>265.0689</td>
<td>16.28094</td>
</tr>
</tbody>
</table>

n = 6
RMSE = 13.60235

GeoXT Grid Sheet

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>27</td>
</tr>
<tr>
<td>25</td>
<td>30.5</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>26</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>112</td>
<td>93.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>18.66667</th>
<th>15.58333</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.33667</td>
<td>10.58333</td>
</tr>
</tbody>
</table>

1D Sums of Distance Error


ROVER
Geodesy & Maps

**Geodesy** – Science of measuring the shape of the Earth

Three important surfaces: **Earth Surface, Geoid, & Ellipsoid**

- Spatial differences in Earth’s density $\Rightarrow$ unequal pull of Gravity
- **GEOID** $\Rightarrow$ Interpolated surface of constant gravitational pull
  - Related to MSL & is a reference for height measures
- **Ellipsoid** $\Rightarrow$ Used to model Earth’s weird shape

**Problems in Defining a Coordinate System:**

1) **Map Projections** – transforming coordinates/locations...
   - Curved Earth to a flat Map $\Rightarrow$ Distortion of Geometry
   - Straight lines & shapes (*i.e.*, continents) $\Rightarrow$ Bent & Distorted

2) **Irregular Earth Surface** $\Rightarrow$ Need a shape that can be modeled
   - Not a round Sphere (Earth isn’t round)
   - Not a perfect elliptical Spheroid either
   - **Ellipsoid** - Special class of Spheroid $\Rightarrow$ *Oblate Spheroid*
Ellipsoids have two defining shape parameters:

- Semi-major axis \((r_1)\)
- Semi-minor axis \((r_2)\)

**Estimates of Ellipsoid Shape**

- **Ancient Past** – **Eratosthenes** (smart Greek guy) over 2200 year ago
  - Estimates circumference or earth \(40,250\text{km}\) \((Eq.\ Circ.: 40,075.16 \Delta 175\text{km})\)
- **1700’s to early 1900’s**
  - Ellipsoid shape estimates varied by country & continent (e.g., Clarke 1866)
  - Due to geoidal variation in Earth’s shape \(\Rightarrow\) Local fits not good elsewhere
  - And, there was no way to integrate mapping surveys across oceans
- **Now, globally optimal ellipsoid shape estimates possible**
  - **WGS72, GRS80, WGS84**
  - Satellite (GRACE), laser, & broadcast signal timing measurements
- **With size & shape of reference ellipsoid determined...**
  - **Earth poles** (axis of ellipsoid rotation) & **Equator** are also defined

**Flattening Factor**

\[
\text{Flattening Factor} \quad f = \frac{r_1 - r_2}{r_1}
\]
Geographic Coordinates

With the Poles & Equator defined...

• We can define a set of Geographic Coordinates
• Reference system to specify positions of features on the Ellipsoid surface

  • Lines of Longitude → Meridians →
    • All converge at poles → unequal E-W distances with Δ in LAT
    • LON circles all centered at center of Earth (Great Circles)
    • 0 = Prime Meridian
    • West LON 0-180°, East LON 0-180°

  • Lines of Latitude → Parallels
    • Do not converge → equal distance N-S
    • Only 0°LAT (Equator) is a Great Circle
    • North LAT 0-90°, South LAT 0-90°

• Because LON distance shrinks from Equator (111.3 km) to pole (0 km)...
  • Features appear distorted when displayed on Cartesian Systems
Datums

So, now that we have an ellipsoid to put LAT LON coordinates on...

We only really know one precise LON ➔ Prime Meridian in Greenwich, England

We must estimate the LAT and LON of all other locations by surveying measurements
  • Observations of stars
  • Measuring distances & directions between points

What the National Geodetic Survey (NGS) does routinely & accurately
  • National network of highly precise, fixed/monumented, survey points
  • Used to establish LAT & LON positions for new ellipsoids
  • Collectively, these points form basis for a GEODETIC DATUM

A Datum is simply a reference surface with two MAIN components
  • Specific ellipsoid w/ a 3D Cartesian Coords (or spherical) and an ORIGIN
  • Set of precise points & lines w/ location estimates for each point in the datum

When new ellipsoids are define (for one reason or another)...
  • All points in the network are resurveyed ➔ called a “realization”
  • In doing so, a new DATUM is created
  • A network from which all other (non-monumented) point can be determined
    NAD27 ➔ bad continent-based ellipsoid (Clarke 1866) & bad realization
    NAD83 ➔ good global-based ellipsoid (GRS80) & excellent realization
In the past, measuring angles was far more accurate than distance measures.

LAT/LON positions were determined by...
- Long-distance sighting (Bilby Towers, mountain peaks, etc)
- Precise angle/azimuth measurements
- Trigonometry

NAD27 → 26,000 pts
NAD83 → 250,000 pts

New Datum
Datum Shifts & Conversions

Mathematically difficult translating between old continental & new global datums
USGS guy: Warren Dewhurst → NADCON 1990

Newer WGS84 Datum
Millions of satellite-based position fixes, but not tied to earth like NAD83
While WGS84 and GRS80 ellipsoid (→ NAD83) are very similar...
...still must convert WGS84 coords to NAD83 before GIS ingest

High Accuracy Reference Network (HARN) → NAD83(HARN)
High Precision Geodetic Networks (HPGN) → NAD83(HPGN) → SAME
Map Projections

The GLOBE

ADVANTAGES
• Directions True
• Distances True
• Shapes True
• Area True

DISADVANTAGES
• Even the largest globe has a very small scale and shows relatively little detail.
• Costly to reproduce and update.
• Difficult to carry around.
• Bulky to store.

FACTS
• Parallels are parallel & equally spaced on meridians
• Meridians (& arcs of Great Circles) are straight lines (If viewed \( \perp \) to Earth’s surface)
• Meridians equally spaced on parallels, but converge at poles & diverge toward the equator
• Meridian spacing = Parallel spacing at Equator
• Meridians @ 60° are \( \frac{1}{2} \) as far apart as parallels

Great circles
• Disks with centers at Earth’s center
• The shortest distance between any two points on the surface of the Earth can be found quickly and easily along a great circle.

Rhumb Lines (loxodromes)
• Lines crossing all meridian of LON @ same angle
• Walking along a fixed bearing (e.g., N45°E)
Great Circle

- Shortest distance between two points on Earth’s surface.
- Any slice (by a plane) through a sphere that intersects the sphere’s center point.
- Equator & all lines of Longitude are Great Circles.

Great Circle Distance

Consider two points on the Earth’s surface, A with geographic coordinates (lat.,lon.), \((\phi_A, \lambda_A)\), and B, with geographic coordinates \((\phi_B, \lambda_B)\).

The great circle distance from point A to point B is given by the formula:

\[
d = r \cdot \cos^{-1}\left[\cos(\phi_A)\cos(\phi_B)\cos(\lambda_A - \lambda_B) + \sin(\phi_A)\sin(\phi_B)\right],
\]

where \(d\) is the shortest distance on the surface of the Earth from A to B, and \(r\) is the Earth’s radius, approximately 6378 km.

This formula may be used to find the distance distortion caused by a projection between two points, for example, between Ursine and Moab, Utah, when using UTM Zone 12N coordinates, NAD83?

Great circle distance:

Latitude, longitude of Ursine, Utah = 37.98481°, -114.216944°
Latitude, longitude of Moab, Utah = 38.57361°, -109.551111°

\[
d = 6378 \cdot \cos^{-1}\left[\cos(37.98481)\cos(38.57361)\cos(-114.216944 - 109.551111) + \sin(37.98481)\sin(38.57361)\right]
\]

= 412,906 km

Grid distance (UTM Zone 12N coordinates):

Grid coordinates of Ursine, Utah = 217,529.8, 4,208,972.8
Grid coordinates of Moab, Utah = 626,239.2, 4,270,405.9

\[
d_g = \left[(X_A - X_B)^2 + (Y_A - Y_B)^2\right]^{0.5}
\]

= \left[(217,529.8 - 626,239.2)^2 + (4,208,972.8 - 4,270,405.9)^2\right]^{0.5}

= 413,300 km

distortion is 412,906 - 413,300 = 404 km, or a 394 meter lengthening.
Rhumb Line (Loxodrome)

Line of constant bearing (e.g., 292.5°)